Reading and Writing to Learn in Mathematics: Strategies to Improve Problem Solving

by Clare Heidema

Reading and Writing in Mathematics

Reading and writing in mathematics are of particular interest to educators because these processes are essential to both problem solving and concept development in mathematics. Martinez and Martinez (2001) discuss what happens when children read and write mathematics:

> For starters, their learning incorporates some key ideas in the National Council of Teachers of Mathematics new *Principles and Standards for School Mathematics* (NCTM, 2000). They learn to use language to focus on and work through problems, to communicate ideas coherently and clearly, to organize ideas and structure arguments, to extend their thinking and knowledge to encompass other perspectives and experiences, to understand their own problem-solving and thinking processes as well as those of others, and to develop flexibility in representing and interpreting ideas. (p. 5)

Reading research clearly points to several characteristics of effective readers. They can:

- Locate key information
- Distinguish between main ideas and supporting details
- Modify their reading behaviors when faced with difficulty
- Ask questions before, during, and after reading
- Construct meaning as they read by monitoring comprehension, evaluating new information, connecting new information with existing ideas, and organizing information in ways that make sense

These characteristics also describe effective readers of mathematics and skills needed to be mathematics problem solvers.

Reading and Writing Strategies for Problem Solving

Mathematics is about problem solving, and reading comprehension is an important component, especially for word problems. Writing, too, is a critical component, because students should monitor and reflect on the problem-solving process as well as communicate their thinking during problem solving.

Problem solving in mathematics often is viewed with a conceptual model proposed by George Polya (1957). Polya's model has four steps:

1. **Understand the problem.** Determine what information is given, what is the unknown, what information is needed or not needed, and what is the context or conditions of the problem. Restate the problem to make sure terminology and facts are understood.

2. **Devise a plan.** Consider how to go about solving the problem and what strategies would help in finding a solution. This may be as simple as selecting the numbers and operations demanded by the problem. It might include examining different ways to approach the problem, for example, comparing it to problems solved previously, or finding related problems, or making and checking predictions.

3. **Carry out the plan.** Use the plan as devised, and check or prove that each action taken is correct.

4. **Look back (and forward).** Examine the result or solution to make sure it is reasonable and solves the problem. Ask if there could be other solutions or if there are other ways to get a solution. Perhaps extend or generalize the problem.

Here is a quick preview of the strategies discussed in this article and how you might use them. The strategies are associated with Polya's four-step model; i.e., each strategy includes the steps in Polya's model.

- **K-N-W-S** and **SQRQCQ**. These are especially useful for helping students understand the steps in problem solving
- **Three-level guide.** This is a good choice for focusing on important facts or approaches.
- **Word problem roulette.** This strategy lends itself nicely to collaboration.
- **Process log and RAFT.** These two strategies are well suited to help students communicate their thinking.

For these reading and writing strategies to become an effective part of a student's "toolbox," teachers must provide instruction on how and when to use them. When providing instruction, consider the following teaching suggestions:

- Introduce one strategy at a time, and let students apply it several times while you observe what they are doing and where they may need help.
- Model and explain the use of a strategy in an activity that lets students see how and why to use it.
- Practice a strategy as a whole class before asking students to use it independently. During the whole-class activity, solicit and compare various responses on how the strategy can work.

K–N–W–S

Description of K-N-W-S

K-W-L (Ogle, 1986) is an active reading tool to help students build content knowledge by focusing on the topic and setting the purpose for the upcoming reading. During K-W-L, students list what they *know* about a topic (K), note questions they have and what they *want* to learn (W), and summarize what they *learned* (L). In a similar pattern, K-N-W-S allows students to use word problems to determine what facts they know (K), what information is *not* relevant (N), what the problem wants them to find out (W), and what *strategy* can be used to solve the problem (S).

In reading, the K-W-L strategy helps all students, no matter the age or achievement level, activate their prior knowledge, develop a purpose for the reading, and make connections between new information and familiar ideas. The K-N-W-S strategy does this as well. In addition, the K-N-W-S strategy allows students to plan, organize, and analyze how to solve word problems, while teachers can evaluate students' understanding and possible misconceptions about word problems. The strategy allows students to focus on what effective students do when assigned word problems.

Guidelines for Use of K-N-W-S

1. Draw a four-column chart on the board or chart paper. Hand out individual charts to students, or have students construct their own.

2. Using a word problem, model how the columns are used. Explain how you know which pieces of information belong in each area of the chart.

3. Students can work in groups or individually to complete K-N-W-S sheets for other word problems. Students can also be asked to write their reasoning for the placement of items in each column.

Example of K-N-W-S

Video Heaven rents movies for $3 a piece per night. The store also offers a video club plan. The plan costs $100 per year and allows unlimited rentals at $1 per movie per night plus two free rentals per month. How many movies must you rent in a year to make the video club worthwhile?

K	N	W	S
What facts do I KNOW from the information in the problem?	**What information do I NOT need?**	**What does the problem WANT to find?**	**What STRATEGY or operations will I use to solve the problem?**
$3 to rent 1 movie. Club plan charges $100/year. Each movie under the plan costs $1. There are 2 free movies per month under the plan.	The video club plan allows an unlimited number of movie rentals.	How many movies must be rented in a year to make joining the club worthwhile?	Make a chart to compare the costs of both regular service and the club plan. Write an inequality to compare the cost of the regular service to the cost of the club plan.

SQRQCQ

Description of SQRQCQ

The reading strategy SQ3R (*survey, question, read, recite, review*) was originated by Robinson (1961) as an independent study tool. SQ3R is based on the idea that if students are to recall and comprehend difficult text, they need to make a conscious effort and be engaged at each stage of the reading process. That idea is also an essential element of SQRQCQ, a six-step study strategy—*survey, question, read, question*, *compute* (construct), *question*—that was designed by Fay (1965) and was modeled after SQ3R. SQRQCQ is intended to assist students in reading and learning mathematics, in particular, solving word problems in mathematics. It allows students to organize in a logical order the steps necessary to solve a word problem.

This strategy can help students focus on a process to decide what a problem is asking, what information is needed, and what approach to use in solving the problem. It also asks students to reflect on what they are doing to solve the problem, on their understanding, and on the reasonableness of a solution.

Guidelines for Use of SQRQCQ

Give students a description of the steps for SQRQCQ. Then model the strategy with one or two word problems before asking students to practice it with other word problems.

1. **Survey.** Skim the problem to get an idea or general understanding of the nature of the problem.
2. **Question.** Ask what the problem is about; what information does it require? Change the wording of the problem into a question, or restate the problem.
3. **Read.** Read the problem carefully (may read aloud) to identify important information, facts, relationships, and details needed to solve the problem. Highlight important information.
4. **Question.** Ask what must be done to solve the problem; for example, "What operations need to be performed, with what numbers, and in what order?" Or "What strategies are needed? What is given, and what is unknown? What are the units?"
5. **Compute (or construct).** Do the computation to solve the problem, or construct a solution by drawing a diagram, making a table, or setting up and solving an equation.
6. **Question.** Ask if the method of solution seems to be correct and the answer reasonable. For example, "Were the calculations done correctly? Were the facts in the problem used correctly? Does the solution make sense? Are the units correct?"

Example of SQRQCQ

Marcie has 96 counters in three colors, red, blue, and yellow. She has twice as many red counters as blue and five times as many blue as yellow. How many counters of each color does Marcie have?

Survey Scan the problem to get a general idea of what it's about. Clarify terms.	Counters are in three colors. There are conditions on the 96 counters Marcie has.
Question What is the problem about, and what is the information in the problem?	How many counters of each color does Marcie have?
Read Identify relationships and facts needed to solve problem.	96 counters (red, blue, and yellow) #red is 2x #blue; #blue is 5x #yellow #red + #blue + #yellow = 96 #red = 2x #blue #blue = 5x #yellow
Question What to do? How to solve the problem?	Write equations with variables r, b, and y. Use substitution. Make a table and try numbers, keeping relationships, until the total is 96. <u>#red</u> <u>#blue</u> <u>#yellow</u> <u>total</u> 20 10 2 32
Compute (or construct) Do the calculations or construct a solution.	Algebra: $r + b + y = 96$ $r = 2b$ and $b = 5y$ $2b + b + y = 96$ (sub. $r = 2b$) $2(5y) + 5y + y = 96$ (sub. $b = 5y$) $10y + 5y + y = 16y = 96$ Therefore $y = 6$, $b = 30$, and $r = 60$ Guess and Check: <u>#red</u> <u>#blue</u> <u>#yellow</u> <u>total</u> 20 10 2 32 40 20 4 64 60 30 6 96 Marcie has 60 red counters, 30 blue counters, and 6 yellow counters.
Question Is the algebra correct? Are the calculations correct? Does the solution make sense?	# blue is 30 (Five times as many blue as yellow) # yellow is 6 The total number of counters is 96.

Three-Level Guide

Description of the Three- Level Guide Strategy
A three-level guide is a teacher-constructed graphic organizer that serves as the basis of a strategy that students can use to solve and get a deeper understanding of the word problem-solving process. With the organizer, students must evaluate facts, concepts, rules, mathematical ideas, and possible approaches to solving a given word problem.

Three-level guides for word problems, suggested by Davis and Gerber (1994), are modeled after a reading strategy using study guides devised by Herber (1978) and further developed by Morris and Stewart-Dore (1984) to help students think through the information in texts. Questions or statements in a study guide address three levels of comprehension: literal, interpretive, and applied.

To create a three-level guide for a word problem, teachers first do a process and content analysis. They think through their objectives for the problem, namely, what students should learn from the problem and what students should know and be able to do. Then they identify in the problem the essential information, the mathematical concepts and relationships, inferences from the relationships, and potential student difficulties. The questions or statements in the guide should help students to develop and appreciate three levels of understanding: the given and relevant information identified explicitly in the problem, the relationships or inferences implicit in solving the problem, and the conclusions or applications involved in solving the problem.

Guidelines for Use of the Three-Level Guide Strategy
Construct guides for problems with three parts (levels). Part I includes a set of true or false facts suggested by the information given in the problem. Part II has mathematics concepts, ideas, or rules that might apply to the problem. Part III includes possible methods (e.g., calculations, creating a table or graph) to use to find a solution to the problem.

1. Introduce students to the three-level guide, and explain the kind of statements that are included in each part.

   Part I: Students analyze each fact to decide if it is true or false given the information in the problem, *and* they decide whether this fact can help them to solve the problem.

   Part II: Students indicate which statements apply to solving the problem, that is, identify concepts or rules that are useful for this problem.

   Part III: Students decide which calculations (or methods) might help them in solving the problem.

2. Model for students the use of a three-level guide in solving a problem.

3. Present students with a three-level guide for another problem, and direct them to complete the guide on their own or with a partner. In this step, students analyze information you have included in the guide to determine both its validity and its usefulness in solving the problem.

4. With advanced students, you might select word problems for which the students write three-level guides to share with the class or to exchange with a partner.

Example of a Three-Level Guide

Read the problem. Follow the directions given for each part.

Problem: The markup rate on electronics at Ed's Electronics Mart is 35%. Ed, the store owner, bought 25 Boom speaker systems for a cost of $70 each. What is the selling price of the Boom speaker system at Ed's Electronics Mart?

Part I (Literal Comprehension)
Directions: Read the statements. Check column A if the statement is true according to the information in the problem. Check column B if the information will help solve the problem.

A (true or false?)	B (helpful?)	
________	________	Ed's markup rate is 35%.
________	________	Ed bought 25 Boom speaker systems.
________	________	The markup rate is less than ¼.
________	________	The cost to Ed for a Boom speaker system is $70.
________	________	The selling price of a Boom speaker system is more than $70.

Part II (Interpretive Comprehension)
Directions: Read the statements. Check the ones that state mathematics ideas useful for the problem.

________ Selling price is greater than cost.
________ Markup equals cost times markup rate.
________ Selling price equals cost plus markup.
________ Markup divided by cost equals markup rate.
________ The total cost of the systems is the cost of one system times the number of systems.

Part III (Applied Comprehension)
Directions: Check the calculations that can be used in solving this problem.

________ 0.35 × $70	________ 25 × $70
________ 25 × 35	________ ($70 × 35) ÷ 100
________ 1.35 × $70	________ $70 + (7/20 × $70)

Word Problem Roulette

Description of Word Problem Roulette
The word problem roulette strategy comes from Davis and Gerber's (1994) discussion of content-area strategies for the secondary mathematics classroom. They suggest that students should discuss and write about the content of word problems and their solutions. The word problem roulette strategy is designed to give students an opportunity to collaborate on solving a word problem and then to communicate as a group the thought processes that went into finding a solution to the problem. The group presents its solution to the problem both orally and in writing.

The strategy involves students in a group problem-solving activity. They read a problem and, as a group, decide what the problem is about and what they might do to solve the problem. Students benefit from communicating their own thinking and from hearing how other students think about a problem. They have a chance to try out different ideas and to come to an agreement on a suitable method to solve a problem.

Guidelines for Use of Word Problem Roulette
Choose word problems that are well suited to collaborative work.

1. Organize the class into cooperative groups of three or four students per group. Provide each group member with a copy of the word problem for the group. Explain to students that they are to solve this problem as a group.

2. First the group discusses how to solve the word problem. The group members talk to one another about what the problem is asking and their ideas for solving the problem, but they do this without writing or drawing on paper. During this step, the members of the group agree on a solution method and the steps for how they will solve the problem.

3. When the group members have agreed on a solution to the problem, they take turns writing the steps to the solution in words rather than mathematics symbols. Each group member writes one step or sentence and then passes the group solution paper to the next group member to add the next step or sentence. Group members may confer on what individual members write, but the solution paper should have contributions from everyone in the group.

4. After all the groups have finished writing their solution papers, choose one group at a time to present its solution to the class. One member of a group reads the solution steps as they are written on the paper, and another group member writes the symbolic representation of this solution on the board.

5. After all the groups who have the same problem have presented their solutions, compare the methods and results of the different groups. If groups have different problems, volunteers from other groups may give an immediate review of a group's solution.

Example of Problem for Word Problem Roulette

Directions:

Read the problem, and discuss a solution with your group.
Do not do any writing during the discussion. When the group agrees on a solution and method to get the solution, write a group report explaining the solution. Each person writes one sentence or step and then passes the paper to another person to do the same. Use mostly words (not symbols) in the report. Everyone contributes in writing the report.

Problem:

A family of three adults and four children goes to an amusement park. Adult admission is twice as much as a child's admission. The family spends $80 on admissions. How much is an adult admission? How much is a child's admission?

Process Logs

Description of Process Logs
Martinez and Martinez (2001) suggest what they call a process log for word problems. This strategy uses a *writing-math* worksheet in which students explain the word problem and the steps they will use to solve it. The worksheet guides the student with question prompts that lead them through the problem-solving process without dictating a method or the steps students use to solve the problem. Students are asked to use ordinary language as well as mathematical language in their explanations.

Using a process log, students write about their thinking during the problem-solving process. The questions they answer create a dialogue between what they know and what they are learning by doing the problem. In this way, students clarify their thinking about the problem and the mathematics involved, they translate mathematical ideas and procedures into ordinary language, and they practice communicating about mathematics and reasoning in problem solving. Process logs are a type of learning log and have the advantage of reinforcing strategic learning processes that we associate with learning logs.

Guidelines for Use of Process Logs
Prepare a writing-math worksheet for students with a word problem activity and an extra challenge. You can include additional question prompts as you consider other ideas you would like in a student's log. You might give students these directions on how to use this worksheet:

1. Use the writing-math worksheet as you think about and solve this problem. That is, "think aloud" on paper about the problem activity.

2. Try to personalize your explanations by writing in the first person; use *I, my, me,* and so on.

3. Use ordinary language as well as mathematical language.

4. Explain the problem itself as you write the steps involved.

5. Be sure to describe any special difficulties with the problem.

6. Explain your problem-solving process step by step.

Example of Process Log

Problem Activity	
Christy works for 2 hours and 15 minutes each week doing yard work. She gets paid $3 an hour for this work. How many weeks will it take Christy to earn enough money to buy a jacket that costs $36.	
Extra Challenge: How much would Christy need to get paid per hour if she wanted to buy the jacket in four weeks?	
	Writing About Problem Solving
How many steps are involved in the problem?	There are two steps: I need to find (1) how many hours Christy must work and (2) how many weeks of work give her this number of hours.
What mathematics operations will you use?	Division (36 ÷ 3) and multiplication. Some number × 2 hours 15 minutes.
Does the problem have special difficulties, things you have to watch out for?	I have to think about the time. 15 minutes is ¼ hour. This will make the multiplication problem easier.
What do you do first?	$36 is the goal for Christy. At $3 per hour she has to work 12 hours (36 ÷ 3 = 12).
Then what...?	Each week she works 2¼ hours. I need to find a number (# weeks) to multiply times 2¼ so the result is equal to or more than 12.
Then what...?	I'll try 5. 5 × 2¼ = 11¼. 5 is too small.
Then what...?	It must be 6. 6 × 2¼ = 13½. In 13½ hours of work, Christy will make more than $36, enough to buy the jacket.
What do you do to check your work?	I'll find how much Christy makes each week and then see how much she makes in 6 weeks. Each week: $3 × 2¼ = $6.75 6 weeks: 6 × $6.75 = $40.50 (more than $36) 5 weeks: 5 × $6.75 = $33.75 (not enough)
How about the extra challenge?	To make $36 in four weeks, she would have to earn $9 each week. At $4 per hour, she would earn $9 each week (4 × 2¼ = 9).

RAFT

Description of RAFT

RAFT (Santa, 1988) is an acronym for the four critical pieces of writing: *role, audience, format,* and *topic.* It is an extended writing activity that expands the topics students have been studying. The RAFT strategy is an excellent way to involve students in explaining what they know about a topic. RAFT can be used as a culminating activity or assessment after students have studied a concept. Options for the students are provided in each of the four areas in order for the students to demonstrate their understanding in a nontraditional format.

Reading research states that students need and appreciate choices in their learning. The choices in the RAFT strategy affect the vocabulary, style, and focus of the writing and address the various learning styles. Students are encouraged to think creatively about the concepts they have been learning. They make connections and internalize ideas when they formulate concepts in a different mode and use them to do further investigations.

The RAFT strategy provides a method for students to synthesize information into a writing-to-learn episode. The National Council of Teachers of Mathematics encourages activities to interconnect mathematical ideas in order to produce a coherent whole. The RAFT strategy allows students to apply mathematics concepts in a fun way to explain what they have learned about a topic.

Guidelines for Use of RAFT

1. Develop a list or brainstorm with students choices for:
   - **Role** of the writer (reporter, observer, eyewitness)
   - **Audience** for the writing (teacher, other students, parent, someone in the community)
   - **Format** to present the writing (letter, article, poem, diary, journal, instructions, advertisement, speech)
   - **Topic** for the writing (application of a procedure, reaction to an event, explanation of a mathematics concept)
2. Each student chooses a role, audience, format, and topic from the generated list. You may assign the same role to all students or let students choose from several different roles.
3. Provide class time for the work. You can conference with students and keep track of their progress on their RAFT choices.
4. Sharing the writing is important. Students can read to the entire class or read to smaller groups, if time is limited.

Example of RAFT

Choose a role to communicate a mathematics topic to someone else. Be as thorough as possible in explaining the topic in the given format. If you want to change the format and audience, please ask. The roles and topic will stay the same.

Roles	Audience	Format	Topic
Zero	Whole numbers	Campaign speech	Importance of 0
Percent	Customers	Advertisement	Mental ways to calculate
Prime number	Jury	Instructions	Rules of divisibility
Container	Self	Diary	Volume measurements

Our Goals for Students

Principles and Standards calls for excellence in mathematics for all students, and this requires high expectations for all students. Adding to or fine-tuning reading and writing strategies in our collection of teaching strategies can help us work with all students to attain these high expectations.

As mathematics teachers, our major goal for students is to learn the mathematics content: to retain important information, develop a deep understanding of concepts and topics, and apply and demonstrate knowledge. In addition, we want students to become independent learners: to gain control of their own learning, ask their own questions, find their own answers, and use critical thinking skills. The use of reading and writing strategies for word problems in mathematics gives us tools with which to help students become more effective readers and communicators in mathematics—or to put it anotherway, these strategies help us address our goals for all students.

Bibliography

Davis, S. J., & Gerber, R. (1994, September). Open to suggestion: Content area strategies in secondary mathematics classrooms. *Journal of Reading, 38*(1), 55–57.

Fay, L. (1965). Reading study skills: Math and science. In J. A. Figural (Ed.), *Reading and inquiry* (pp. 93–94). Newark, DE: International Reading Association.

Herber, H. (1978). *Teaching reading in the content areas* (2nd ed.). Englewood Cliffs, NJ: Prentice Hall.

Martinez, J. G. R., & Martinez, N. C. (2001). *Reading and writing to learn mathematics: A guide and a resource book.* Boston: Allyn & Bacon.

Morris, A., & Stewart-Dore, N. (1984). *Learning to learn from text: Effective reading in content areas.* New South Wales: Addison-Wesley.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Reading Panel. (2000). *Teaching children to read: An evidence-based assessment of the scientific research literature on reading and its implications for reading instruction.* Washington, DC: National Institutes of Health.

Ogle, D. (1986, February). The K-W-L: A teaching model that develops active reading of expository text. *The Reading Teacher, 39*, 564–570.

Polya, G. (1957). *How to solve it: A new aspect of mathematical method* (2nd ed.). Princeton, NJ: Princeton University Press.

Robinson, F. (1961). *Effective study.* New York: Harper & Row.

Santa, C. M. (1988). *Content reading including study systems.* Dubuque, IA: Kendall-Hunt.

Clare Heidema is a senior research associate at RMC Research Corporation in Denver, Colorado. She has teaching experience at all levels from elementary school to graduate school, has extensive background in professional development, and has written curriculum materials such as the Comprehensive School Mathematics Program. She is coauthor of Teaching Reading in Mathematics: A Supplement to Teaching Reading in the Content Areas Teacher's Manual.